NONPERTURBATIVE EFFECTS IN QCD AT $T > 0$

Yu.A.Simonov
Institute of Theoretical and Experimental Physics
Moscow, 117259

Abstract

The background field formalism is used to implement nonperturbative QCD contributions into diagrammatic technic at $T > 0$. The leading terms both in the confining and nonconfining phase are identified at large N_c and the transition temperature is calculated in this limit, which appears to be in good agreement with the Monte Carlo calculations.

1 Introduction

It is believed that the perturbative QCD is applicable in the deconfined phase at large enough temperatures T, where the effective coupling constant $g(T)$ is small [1], while at small T (in the confined phase) the nonperturbative effects instead are most important. However even at large T the physics is not that simple: some effects, like screening (electric gluon mass), need a resummation of the perturbative series [2], while the effects connected with the magnetic gluon mass demonstrate the infrared divergence of the series [3].

There is a substantial amount of data from lattice calculations in the deconfined phase, which can be explained only by nonperturbative effects (see discussion below in section 6). Therefore one can assume that nonperturbative effects are of vital importance both for small and large T and should be taken into account before any dynamical perturbative scheme is

applied. We suggest to use for this purpose the background field formalism [4], in connection with the cluster expansion for background fields [5] and the Feynman-Schwinger representation (FSR) [6] for the Green's function.

This method has been used successfully for QCD at $T = 0$ and it contains confinement as an essential ingredient, appearing through the area law of Wilson loops in FSR.

It is a purpose of this paper to apply the method to the case of $T > 0$ by proper modifications of FSR and the background field method (BFM). For this purpose we write in Section 2 the familiar BFM expressions for the free energy with the only important new element: we do not consider the background field as classical (as it is usually done) but rather make an average over an ensemble of background fields, so that the latter enter final expressions for the free energy in the form of gauge-invariant correlators.

As a consequence the simple loop diagram– the leading term $0(g^0)$ of the usual perturbation theory — becomes a set of diagrams representing multiple loops interacting via nonperturbative correlators. This is done in Section 3 for gluons and in Section 4 for quarks.

The important new element there is a new form of FSR for a gluon or a quark Green's function appropriate for $T > 0$: it contains a path integral winding on a torus which compactifies the fourth Euclidean direction. This enables us to compute gauge-invariant Green's function for hadrons and response functions for $T > 0$.

Higher order amplitudes are discussed in Section 5. Here the behaviour of the QCD coupling constant g in strong background field at all distances is exploited [7].

In Section 6 we identify the leading contributions to the free energy in the limit when $N_c \to \infty$ and g is small. It is gratifying that the phase transition of the first order occurs already in this oversimplifying case and the resulting transition temperature T_c does not depend on N_c when N_c is large.

Clearly the mechanism of the phase transition is the evaporation of a part of the gluonic condensate into a gas of almost free gluons [8]. The predicted value of T_c agrees well with Monte Carlo calculations both in the case of gluodynamics [9] and in the case of QCD [10] for different number of flavours.

In conclusion possible prospectives of the proposed method are outlined.

2 Basic equations

We start with standard formulas of the background field formalism [4] generalized to the case of nonzero temperature. We assume that the gluonic field A_μ can be split into the background field B_μ and the quantum field a_μ

$$A_\mu = B_\mu + a_\mu, \tag{1}$$

both satisfying periodic boundary conditions

$$B_\mu(z_4, z_i) = B_\mu(z_4 + n\beta, z_i), a_\mu(z_4, z_i) = a_\mu(z_4 + n\beta, z_i), \tag{2}$$

where n is an integer and $\beta = 1/T$.

The partition function can be written as

$$Z(V, T, \mu = 0) =< Z(B) >_B ,$$

$$Z(B) = N \int D\phi exp(-\int_0^\beta d\tau \int d^3x L(x, \tau)) \tag{3}$$

where ϕ denotes all set of fields a_μ, Ψ, Ψ^+, N is a normalization constant, and the sign $<>_B$ means some averaging over (nonperturbative) background fields B_μ, the exact form of this averaging is not needed for our purposes. Furthermore, we have

$$L(x, \tau) = \sum_{i=1}^{8} L_i,$$

where

$$\begin{aligned} L_1 = \frac{1}{4}(F^a_{\mu\nu}(B))^2; L_2 = \frac{1}{2} a^a_\mu W^{ab}_{\mu\nu} a^b_\nu, \\ L_3 = \bar{\Theta}^a (D^2(B))_{ab} \Theta^b; L_4 = -ig\bar{\Theta}^a (D_\mu, a_\mu)_{ab} \Theta^b \\ L_5 = \frac{1}{2}\alpha(D_\mu(B)a_\mu)^2; L_6 = L_{int}(a^3, a^4) \\ L_7 = -a_\nu D_\mu(B) F_{\mu\nu}(B); L_8 = \Psi^+(m + \hat{D}(B + a))\Psi \end{aligned} \tag{4}$$

Here $\bar{\Theta}, \Theta$ are ghost fields, α- gauge–fixing constant, L_6 contains 3–gluon– and 4–gluon vertices, and we keep the most general background field B_μ, not satisfying classical equations, hence the the appearence of L_7.

The inverse gluon propagator in the background gauge is

$$W^{ab}_{\mu\nu} = -D^2(B)_{ab} \cdot \delta_{\mu\nu} - 2gF^c_{\mu\nu}(B) f^{acb} \tag{5}$$

where

$$(D_\lambda)_{ca} = \partial_\lambda \delta_{ca} - igT^b_{ca} B^b_\lambda \equiv \partial_\lambda \delta_{ca} - g f_{bca} B^b_\lambda \tag{6}$$

We cosider first the case of pure gluodinamics, $L_8 \equiv 0$.

Integration over ghost and gluon degrees of freedom in (3) yields

$$Z(B) = N'(detW(B))^{-1/2}_{reg} [det(-D_\mu(B) D_\mu(B+a))]_{a=\frac{\delta}{\delta J}} \times$$
$$\times \{1 + \sum^\infty_{l=1} \frac{S^l_{int}}{l!} (a = \frac{\delta}{\delta J})\} exp(-\frac{1}{2} JW^{-1}J)_{J_\mu = \quad D_\mu(B) F_{\mu\nu}(B)} \tag{7}$$

One can consider strong background fields, so that gB_μ is large (as compared to Λ^2_{QCD}), while $\alpha_s = \frac{g^2}{4\pi}$ in that strong background is small at all distances [7].

In this case Eq. (7) is a perturbative sum in powers of g^n, arising from expansion in $(ga_\mu)^n$.

In what follows we shall discuss the Feynman graphs for the free energy $F(T)$, connected to $Z(B)$ via

$$F(T) = -Tln < Z(B) >_B \tag{8}$$

As will be seen, the lowest order graphs already contain a nontrivial dynamical mechanism for the deconfinement transition, and those will be considered in the next section.

3 The lowest order gluon contribution

To the lowest order in ga_μ (and keeping all dependence on gB_μ explicitly) one has

$$Z_0 = e^{-F_0(T)/T} = N' < exp(-F_0(B)/T) >_B \tag{9}$$

where using (7) $F_0(B)$ can be written as

$$\frac{1}{T} F_0(B) = \frac{1}{2} lndetW - lndet(-D^2(B)) =$$
$$= Sp\{-\frac{1}{2} \int^\infty_0 \zeta(t) \frac{dt}{t} e^{-tW} + \int^\infty_0 \zeta(t) \frac{dt}{t} e^{tD^2(B)}\} \tag{10}$$

In (10) Sp implies summing over all variables (Lorentz and color indices and coordinates), $\zeta(t) = lim \frac{d}{ds} \frac{M^{2s}t^s}{\Gamma(s)} \mid_{s=0}$ is a regularizing factor, one can use also the Pauli-Villars form for $\zeta(t)$.

Graphically the first term on the r.h.s. of (10) is shown in Fig.1 and is a gluon loop in the background field, the second term is a ghost loop, shown in Fig.2.

Let us turn now to the averaging procedure in (9). With the notation $\varphi = -F_0(B)/T$, one can exploit in (9) the cluster expansion [5]

$$< exp\varphi >_B = exp \sum_{n=1}^{\infty} \ll \varphi^n \gg \frac{1}{n!} =$$
$$exp\{< \varphi >_B + \frac{1}{2}[< \varphi^2 >_B - < \varphi >_B^2] + 0(\varphi^3)\}. \tag{11}$$

To get a closer look at $< \varphi >_B$ we first should discuss termal propagators of gluon and ghost in the background field. We start with the thermal ghost propagator and write the FSR for it [6]

$$(-D^2)^{-1}_{xy} = < x \mid \int_0^\infty dt e^{tD^2(B)} \mid y > = \int_0^\infty dt (Dz)^w_{xy} e^{-K} \hat{\Phi}(x,y) \tag{12}$$

Here $\hat{\Phi}$ is the parallel transporter in the adjoint representation along the trajectory of the ghost:

$$\hat{\Phi}(x,y) = Pexp(ig \int \hat{B}_\mu(z) dz_\mu), \tag{13}$$

also

$$K = \frac{1}{4} \int_0^t d\tau \dot{z}^2_\mu; \quad \dot{z}_\mu = \frac{\partial z_\mu(\tau)}{\partial \tau}$$

and $(Dz)^w_{xy}$ is a path integration with boundary conditions imbedded (this is marked by the subscript (xy)) and with all possible windings in the Euclidean temporal direction.

One can write it explicitly as

$$(Dz)^w_{xy} = \prod_{m=1}^N \frac{d^4\zeta(m)}{(4\pi\varepsilon)^2} \sum_{n=0,\pm,\ldots} \frac{d^4p}{(2\pi)^4} e^{ip(\sum_{m=1}^N \zeta(m) - (x-y) - n\beta\delta_{\mu 4})} \tag{14}$$

Here $\zeta(k) = z(k) - z(k-1), N\varepsilon = t$.

One can check that in the free case, $\hat{B}_\mu = 0$, Eq.(12) reduces to well-known form of the free propagator

$$(-\partial^2)^{-1}_{xy} = \int_0^\infty dt e^{-\sum_1^N \frac{\zeta^2(m)}{4\varepsilon}} \prod_m \overline{d\zeta(m)} \sum_n \frac{d^4p}{(2\pi)^4} e^{ip(\Sigma\zeta(m)-(x-y)-n\beta\delta_{\mu 4})} =$$

$$= \sum_n \int_0^\infty e^{-p^2t-ip(x-z)-ip_4n\beta} dt \frac{d^4p}{(2\pi)^4} \quad (15)$$

with

$$\overline{d\zeta(m)} \equiv \frac{d\zeta(m)}{(4\pi\varepsilon)^2}.$$

Using the Poisson summation formula

$$\frac{1}{2\pi} \sum_{n=0,\pm1,\pm2\dots} exp(ip_4n\beta) = \sum_{k=0,\pm1,\dots} \delta(p_4\beta - 2\pi k) \quad (16)$$

one finally gets the Jackiw-Templeton's form [1,2]

$$(-\partial^2)^{-1}_{xy} = \sum_{k=0,\pm1,\dots} \int \frac{Td^3p}{(2\pi)^3} \frac{e^{-ip_i(x-y)_i - i2\pi kT(x_4-y_4)}}{p_i^2 + (2\pi kT)^2} \quad (17)$$

Note that as expected the propagator (12),(17) corresponds to a sum of ghost paths with all ppossible windings around the torus. The momentum integration in (14) asserts that the sum of all infinitesimal "walks" $\zeta(m)$ should be equal to the distance $(x-y)$ modulo n windings in the conpactified fourth coordinate.

For the gluon propagator in the backgrounvd gauge we obtain similarly to (12)

$$(W)^{-1}_{xy} = \int_0^\infty dt (Dz)^w_{xy} e^{-K} \hat{\Phi}_F(x,y) \quad (18)$$

where

$$\hat{\Phi}_F(x,y) = P_F P exp(-2ig \int_0^t \hat{F}(z(\tau))d\tau) exp ig \int_y^x \hat{B}_\mu dz_\mu \quad (19)$$

and the operators P_FP are used to order insertions of $\hat{F}$ on the trajectory of the gluon [11,12].

Now we come back to the first term in (11), $<\varphi>_B$, which can be represented with the help of (12) and (18) as

$$<\varphi>_B=\int\frac{dt}{t}\zeta(t)d^4x(Dz)^w_{xx}e^{-K}[\frac{1}{2}tr<\hat{\Phi}_F(x,x)>_B-<tr\hat{\Phi}(x,x)>_B] \tag{20}$$

where the sign tr implies summation over Lorentz and color indices.

In the Appendix 1 we show that Eq.(20) yields for $B_\mu=0$ the usual result of the free gluon gas:

$$F_0(B=0)=-T\varphi(B=0)=-(N_c^2-1)V_3\frac{T^4\pi^2}{45} \tag{21}$$

As a next example we consider the case of nonzero component $\hat{B}_4$, while $\hat{B}_i=0, i=1,2,3$. We also neglect for simplicity the difference between $\hat{\Phi}_F$ and $\hat{\Phi}$, i.e. we put $\hat{F}=0$. Actually the interactions of $\hat{F}$ represent the interaction of the gluon spin with the background field [11, 12] and gives rise to short-range correlations (in the confining phase it also yields spin-orbit interaction, the nonperturbative part of which, the Thomas precession, is long range, $\sim 1/r$) with the notation

$$<\hat{\Omega}>_B=<expig\int_0^\beta\hat{B}_4(z)dz_4>_B \tag{22}$$

one obtains from (20)

$$<\varphi>_B=Tr_c\int\frac{dt}{t}\zeta(t)V_3\beta\frac{d^4p}{(2\pi)^4}\sum_{n=0,\pm1}e^{-p^2t-ip_4n\beta}<\hat{\Omega}^n>_B=$$

$$=-Tr_cV_3\int\frac{d^3p}{(2\pi)^3}\int_{\beta p}^M d\Theta(1+2\sum_{n=1,2}e^{-\Theta n}<\hat{\Omega}^n>_B)= \tag{23}$$

$$=Tr_c(-2V_3\int\frac{d^3p}{(2\pi)^3}<ln(1-e^{-\beta p}\hat{\Omega})>_B=$$

$$=Tr_c<\frac{\beta V_3}{3\pi^2}\int_0^\infty\frac{p^3dp}{e^{\beta p}\hat{\Omega}^{-1}-1}>_B=Tr_c\frac{2T^4}{\pi^2}V_3\sum_{n=1,2,...}\frac{<\hat{\Omega}^n>_B+<\hat{\Omega}^{*n}>_B}{2n^4} \tag{24}$$

In the case $\Omega\equiv 1$ we come back to the free case, Eq.(21), since

$$\sum_{n=1}^\infty\frac{1}{n^4}=\frac{\pi^4}{90} \tag{25}$$

The expression (24) for a constant field $\hat{B}_4$ yields the well-known result

$$F_0(\hat{B}_4 = const) = -\frac{2V_3T^4}{\pi^2}Tr_c \sum_{n=1,2,...} \frac{\cos(g\hat{B}_4 n\beta)}{n^4} \tag{26}$$

where Tr_c means trace over color indices.

One can notice in (26), (24) that in general the presence of field $\hat{B}_4$(or $\hat{\Omega} \neq 1$) descreases the modulus of F_0, i.e. is not advantageous from the point of view of the minimum of F_0.

Thus $< \varphi >_B = -F_0/T$ in the deconfined regime represents a loop contribution of the gluon gas in the background field. Now we turn to the confining regime and again calculate $< \varphi >_B$. It is convenient to choose another point y on the loop in addition to the initial point x, as shown in Fig.3 and to write $< \varphi >_B$ as

$$< \varphi >_B = \int_0^\infty \zeta(t)\frac{dt}{t}\int_0^t \frac{dt_1}{t} d^4y d^4x (Dz)^w_{xy}(Dz')^w_{xy} e^{-K-K'-\sigma_a S} \tag{27}$$

where we have introduced the area law for the closed contour average

$$< \hat{\Phi}(x,x) > = exp(-\sigma_a S) \tag{28}$$

It is important at this point to stress that in the confining regime the total contour $C'(x,y,x)$ shown in Fig.4, made by trajectories $z(\tau)$ and $z'(\tau)$ in (27), should be a closed contour, or in other words, in the representation

$$Dz(\tau) = \prod_{m=1}^{N} \overline{d\zeta(m)} \sum_n \frac{d^4p}{(2\pi)^4} exp(ip(\sum \zeta(m) - (x-y) - n\beta\delta_{\mu 4}))$$
$$Dz'(\tau') = \prod_{m'=1}^{N'} \overline{d\zeta(m')} \sum_{n'} \frac{d^4p'}{(2\pi)^4} exp(ip'(\sum \zeta'(m') - (x-y) - n'\beta\delta_{\mu 4})) \tag{29}$$

one should choose

$$n = n' \tag{30}$$

In this case both trajectories $z(\tau)$ and $z'(\tau')$ are winding in the same way on the torus and the minimal surface S lies on the surface of the torus, winding around it n times. In the case $n \neq n'$ there appear a piece of length $\mid n - n' \mid$ where a gluon is winding and contributing an infinite amount to the free energy (since we assume that the free energy of an isolated quark or gluon

is ininite). Thus for $n = n'$ one can integrate in (27) , (29) over $d^4(x-y)$ as follows (note that $x_4 - y_4$ changes in the limits $[0\beta]$)

$$d^4(x-y)\sum_n exp[-i(\vec{p}+\vec{p'})(\vec{x}-\vec{y})-i(p_4+p'_4)(x_4-y_4+n\beta)] = (2\pi)^4\delta^4(p+p') \tag{31}$$

We obtain for $<\varphi>_B$

$$<\varphi>_B = \int \zeta(t)\frac{dt}{t}\frac{dt_1}{t}V_3\beta G(t_1, t-t_1) \tag{32}$$

where we have introduced

$$G(t_1, t-t_1) = \frac{d^4p}{(2\pi)^4}e^{ip\sum\zeta_r}\prod\overline{d\zeta_r d\zeta_R}e^{-\frac{\sum\zeta_R^2(m)}{4\varepsilon}-\frac{\sum\zeta_r^2(m)}{4\varepsilon}-\sigma_a S} \tag{33}$$

$$\text{and} \quad \zeta_r = \zeta(m) - \zeta'(m), \zeta_R = \frac{1}{2}(\zeta(m) + \zeta'(m))$$

One can notice that temperature enters only as a factor β in (32) and it is cancelled in the free energy

$$F_0 = -T <\varphi>_B = -V_3\int \zeta(t)\frac{dt}{t}\frac{dt_1}{t}G(t_1, t-t_1) \tag{34}$$

One can also rewrite G through the two-gluon Green's function $G(xx, 00, t, t-t_1)$, where gluons start at the point 0 and meet at the point x:

$$G(t_1, t-t_1) \equiv \int d^4x G((xx; 00; t_1, t-t_1). \tag{35}$$

In the free case it is equal to $\frac{d^4p}{(2\pi)^4}e^{-p^2t}$. Note however that this result coincides with the loop diagram for $T = 0$, which is entirely absorbed by the subtraction constant in calculating (21).

In case of confinement one obtains instead

$$G = \sum_n \varphi_n^2(0)e^{-M_n^2 t} \tag{36}$$

where $\varphi_n(0), M_n$ are wavefunction at origin and glueball mass respectively.

In any case our result for the gluon loop with confinement, Eq.(34), does not depend on temperature and should be included in the value of the free energy for zero temperature, which as we shall argue below is given by the scale anomaly [14] ,i.e. by the gluonic condensate.

Note that physically this correspondence is quite understandable, since the gluon loop contributimon (34) is expressed through the two-gluon bound state at the total momentum zero, i.e. through the condensate of bound gluon-gluon pairs (glueballs), and the latter is most generally given by the gluonic condensate $< F_{\mu\nu} F_{\mu\nu} >$ via the scale anomaly expression.

4 The lowest order quark contribution

Integrating over quark fields in (3) is done trivially and leads to the following additional factor in (7)

$$det(m + \hat{D}(B + a)) = \frac{1}{2} det(m^2 - \hat{D}^2(B + a)) \qquad (37)$$

where we have used the symmetry property of eigenvalues of $\hat{D}$ In the lowest approximation we omit a_μ in (37) and write the free energy contribution $F_0(B)$ similary to the gluon contribution (9-10) as

$$\frac{1}{T} F_0^q(B) = -\frac{1}{2} lndet(m^2 - \hat{D}^2(B)) = -\frac{1}{2} Sp \int_0^\infty \zeta(t) \frac{dt}{t} e^{-tm^2 + t\hat{D}^2(B)} \qquad (38)$$

where the sign Sp has the same meaning as in (10) and

$$\hat{D}^2 = (D_\mu \gamma_\mu)^2 = D_\mu^2(B) - g F_{\mu\nu} \sigma_{\mu\nu} \equiv D^2 - g\Sigma F; \quad \sigma_{\mu\nu} = +\frac{i}{4}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu); \qquad (39)$$

Our aim now is to exploit the FSR to represent (38) in a form of the path integral, as it was done for gluons in (14). The equivalent form for quarks must implement the antisymmetric boundary conditions pertinent to fermions. It is easy to understand that the correct form for quark is

$$\frac{1}{T} F_0^q(B) = -\frac{1}{2} tr \int_0^\infty \zeta(t) \frac{dt}{t} d^4x \overline{(Dz)}^w_{xx} e^{-K - tm^2} W_{\sum}(C_n) \qquad (40)$$

where $W_{\sum}(C_n) = P_F P_A expig \int_{C_n} A_\mu dz_\mu expg(\sum F)$

$$\overline{(Dz)}^w_{xy} = \prod_{m=1}^N \frac{d^4\zeta(m)}{(4\pi\varepsilon)^2} \sum_{n=0,\pm1,\pm2,\ldots} (-1)^n \frac{d^4p}{(2\pi)^4} e^{ip(\sum_{m=1}^N \zeta(m) - (x-y) - n\beta\delta_{\mu 4}} \qquad (41)$$

One can easily check that in the case $B_\mu = 0$ one is recovering the well-known expression for the free quark gas

$$F_0^q(free\ quark) = -\frac{7\pi^2}{180} N_c V_3 T^4 \cdot n_f, \tag{42}$$

where n_f is the number of flavors. The derivation of (42) starting from the path-integral form(40) is done similarly to the gluon case given in the Appendix 1.

The loop C_n in (40) corresponds to n windings in the fourth direction. Above the deconfinement transition temperature T_c one can visualize in (40) the appearance of the factor

$$\Omega = exp\ ig \int_0^\beta B_4(z) dz_4 \tag{43}$$

(Note the absence of the dash sign in (43) as compared to (22) implying that in (43) B_4 is taken in the fundamental representation). For the constant field B_4 and $B_i = 0, i = 1, 2, 3$ one obtains

$$< F >= -\frac{V_3}{\pi^2} tr_c \sum_{n=1}^{\infty} \frac{\Omega^n + \Omega^{-n}}{n^4} (-1)^{n+1} \tag{44}$$

This result coincides with the obtained in the literature [13].

5 Higher order contributions

We unify under this title two different types of contributions: i) higher order cumulants from the cluster expansion (11) for the determinantal term (9); ii) terms of the higher order in g, appearing in the perturbative expansion (7).

We discuss first the terms i). A generic case is given by the quadratic cumulant $\ll \varphi^2 \gg_B$ which can be written as (we neglect the gluon spin terms)

$$\ll \varphi^2 \gg_B = \int_0^\infty \zeta(t) \frac{dt}{t} \int_0^\infty \zeta(t') \frac{dt'}{t'} d^4x d^4x' e^{-K-K'} (Dz)^w_{xx} (Dz')^w_{x'x'} \times \tag{45}$$

$$\times \ll tr\hat{\Phi}(x,x) tr\hat{\Phi}(x',x') \gg$$

where $\hat{\Phi}$ is defined in (13). The cumulant in the integral (45) corresponds to the interaction between two gluon loops, each loop being a gauge-invariant object. Hence the interaction of the loops, given by the cumulant, is suppressed as $1/N_c^2$ both in the confining and nonconfining regimes(as compared to the contribution $< \varphi >_B$ in (11)).

In the confining regime two gluons form a bound state – a glueball. One can choose the initial glueball state connecting the loops (xx) and $(x'x')$ by the stright line at some fixed $x_4 = x_4'$, and then (45) can be rewritten as

$$\ll \varphi^2 \gg = \int \zeta(t) \frac{dt}{t} \int \zeta(t') \frac{dt'}{t'} V_3^2 \beta^2 e^{-K-K'-\sigma_a S} (DR)^w_{xx} (Dr)^w_{00} \qquad (46)$$

where R and r are central and relative coordinates of two gluons defined in accordance with (33). The resulting integrals in (46) are similar to those in (32) with one essential difference; in (32) the integration over all $d(x-y)$ as in (31) is performed leading to the total momentum of two gluons $p+p'=0$. In contrast to that in (46) the initial and final central point R is the same and therefore all values of total momentum are possible as in the one–gluon case, see (15) and Appendix 1.

As a result one obtains the temperature dependence and the final result (see Appendix B for details) is the free energy of the glueball state [15]

$$\ll \varphi^2 \gg = \sum_k \frac{V_3 (2m_k T)^{3/2}}{8\pi^{3/2}} e^{-m_k/T} \qquad (47)$$

where m_k is the glueball mass in the glueball state k. Note that (47) is $0(N_c^0)$ since it is a color singlet contribution, in contrast to the gluon contribution (21).

In the same way one can consider the next irreducible average, $\ll \varphi^3 \gg$. The contribution is again a color singlet, corresponds to the three-gluon glueball in the confining regime and is of the order $0(N_c^0)$.

The same features are common to higher order cumulants.

In a similar manner one can study the quark contributions to $\ll \varphi^2 \gg_B$. In this case one has to replace in (45) the adjoint parallel transporters $\hat{\Phi}$ by the fundamental ones and use the fermion boundary conditions in $(Dz)^w_{xy}$ as in (41). In the confining regime one obtains the meson contribution to the free energy which has the same form as in (47). It is of the order of (N_c^0).

The largest contribution at small temperatures, $T \ll T_c$ comes from the pion gas, where neglecting the pion mass we have

$$F_0(pion\ gas) = -\frac{\pi^2}{30} V_3 T^4 \tag{48}$$

One should note, that in the set i) introduced above the gluons interact whith each other via nonperturbative field B_μ, which can be exemplified by the correlators $\ll F_{\mu\nu}(B) F_{\lambda\sigma}(B) \gg$ etc. In contrast to that in the set ii) all perturbative exchanges are taken into account. These are proportional to g^n and in the usual treatment $g = g(T)$ is small at large temperature while it may diverge as $T \to \Lambda$ [1]

$$g^2(T) = \frac{24\pi^2}{(11N_c - 2N_f) lnT/\Lambda}, \tag{49}$$

In the strong background field however one has an additional scale parameter $< trF^2_{\mu\nu}(B) > \equiv B$ which defines dynamics both at small and large distances and temperatures. One can show that when $B \gg \Lambda^4$ this scale defines the value of the coupling constant $g(B,T)$ and $g(B,T)$ remains small even when T tends to zero [7]. Therefore one may hope that the perturbative terms (set ii)) which yield relatively small contribution ($\sim 10\%$) at $T \sim 3T_c$, remain of the same order at all temperatures.

6 Calculation of the deconfiniment temperature in the leading order of the $1/N_c$ expansion

In this section we estimate the deconfinement temperature keeping only leading terms in both phases – confining and deconfining. Results given below have been published in a short form in [8]. We start with the zero temperature and note that the NP energy density ε is connected via the scale anomaly [14] to the gluonic condensate [16] (we neglect the quark masses since only light quarks are important at low temperatures)

$$\varepsilon = \frac{\beta(\alpha_s)}{16\alpha_s} < G^a_{\mu\nu} G^a_{\mu\nu} > \cong -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} < G^2 > \tag{50}$$

One can associate with ε the zero–temperature limit of the free (Helmholtz) energy $F(T=0)$ and write F at nonzero temperature as

$$F = \varepsilon V_3 + f(T) \tag{51}$$

where $f(T)$ is to be computed using the formulas (8) and subsequent ones. Since $f(T)$ is regularized by subtracting the zero-temperature limit $f(T=0)$ and all divergencies are present in the latter term, Eq.(51) is a definition of the regularized limit $F(T=0)$ where only NP contribution in the gluonic condensate is present. A check of correctness of our normalization of $F(T=0)$ is given by the computation of the leading contribution in Section 3.

There the perturbative contribution (21) goes to zero as $T \to 0$ in accordance with our prescription that perturbative contributions are normalized to vanish at $T \to 0$. If however one takes into account the confining background, one gets for $T \to 0$ a nonzero contribution (34), which is exactly due to a pair of gluons connected by the adjoint string (28) and with zero total momentum (see (31)).

This is exactly the contribution which one expects from a gluon pair forming a condensate.

The low–temperature phase consists of the gluonic condensate (50), glueballs (47), mesons (we keep only the pion (48)) and their interacting conglomerates:

$$F_{low} = \varepsilon V_3 - T \sum_K \frac{V_3(2m_K T)^{3/2}}{8\pi^{3/2}} e^{m_K/T} - \frac{\pi^2}{30} V_3 T^4 + 0(1/N_c) \tag{52}$$

Here the first term is the leading in N_c; it grows as N_c^2 [17], while the ideal gas of glueballs and mesons contributes $0(N_c^0)$. Note that the interaction between white objects is suppressed at $N_c \to \infty$ [18] and is presented in (52) by the last term.

The high–temperature phase is belived to be the gas of quarks and gluons, interacting perturbatively [1,2]. We shall now argue that i) there is at $T > T_c$ another component of the phase, namely a part of the gluonic condensate, which has not evaporated during the phase transition; ii) there is a NP interaction between quarks and gluons which is important in some situations.

To start we remind that the NP interaction is governed by the correlators [6]

$$G_{\mu\nu,\lambda\sigma} \equiv < tr[F_{\mu\nu}(x)\Phi(x,y)F_{\lambda\sigma}(y)\Phi(y,x)] > \tag{53}$$

which at $T = 0$ contain two independent Lorentz structures with scalar coefficients $D(x-y)$ and $D_1(x-y)$ [5,6]

$$G_{\rho\mu,\sigma\nu}(u) = (\delta_{\rho\sigma}\delta_{\mu\nu} - \delta_{\rho\nu}\delta_{\mu\sigma})D(u) + \frac{1}{2}[\frac{\partial}{\partial u_\rho}((u)_\sigma\delta_{\mu\nu} - (u)_\nu\delta_{\mu\sigma}) + (\rho\sigma \leftrightarrow \mu\nu)]D_1(u) \tag{54}$$

At $T > 0$ the Lorentz invariance is broken and one obtains two independent correlators– for electric and magnetic fields respectively [19]

$$G^{(E)}_{ik} \equiv G_{i4,k4} = \delta_{ik}D^{(E)}(u) + \frac{1}{2}[...]D^{(E)}_1(u) \tag{55}$$

$$G^{(B)}_{ik} \equiv G_{em,pq}\frac{1}{2}e_{iem}\frac{1}{2}e_{kpq} = \delta_{ik}D^B(u) + \frac{1}{2}[...]D^B_1(u) \tag{56}$$

For the gluon condensate one must put $u = 0$ in (54-55) with the result

$$< trF_{\mu\nu}(0)F_{\mu\nu}(0) >= D^E(0) + D^E_1(0) + D^B(0) + D^B_1(0) \tag{57}$$

Out of four functions in (56) only one, D^E, is connected to the confinement, since the string tension σ is expressed through it [5,6]

$$\sigma = \frac{1}{2}\int\int^{\infty}_{-\infty} D^E(\sqrt{u^2_1 + u^2_4})du_1du_4 + ... \tag{58}$$

where dots imply contributions of higher-order cumulants.

Note that the integral (58) is confined to the region $\sqrt{u^2_1 + u^2_4} \preceq T_g$, since D^E falls off exponentially outside. The value of T_g – the gluonic correlation length – is the order of $0,2fm$ [20].

Therefore for the temperatures $T \preceq T_c \simeq 200MeV$ the periodicity in the fourth direction does not change σ significantly, and the deconfinement occurs when D^E disappears at some $T = T_c$ together with σ. There is no reason why D^E_1, D^B, D^B_1 disappear at $T = T_c$ or at $T > T_C$. Moreover, there is evidence [21] that D^B, which defines the area law of the spacial Wilson loops, should be nonzero at $T > T_c$. Additional evidence comes from the hadronic screening lengths [22], also implying the confining dynamics in the spacial directions (we refer the reader to [9,19] for more discussion of this point).

Thus we can assume that only D^E disappears at $T = T_c$. How gluonic condensate changes with temperature? The answer comes from lattice calculations [23] and shows that $< trF^2_{\mu\nu}(0) >$ is rather stable and only a part of

it disappears near $T = T_c$. This is understandable from the theoretical point of view, since the natural scale of $< trF^2(0) >$ is given by the lowest glueball mass approximately equal to the mass of the dilaton [24] which is of the order of $1 Gev$, and therefore $< trF^2(0) >$ should not strongly change in the range $0 \preceq T \preceq T_c$. Thus we come to the following model for high–temperature phase:

$$F_{high} = (1-\eta)\varepsilon V_3 - (N_c^2 - 1)V_3 \frac{T^4\pi^2}{45} - \frac{7\pi^2}{180} N_c V_3 T^4 n_f \tag{59}$$

where

$$\eta = \frac{D^E(0)}{< trF^2(0) >}, \tag{60}$$

Since D_1 enters the NP tensor force of quarkonia [11] one can make some estimate of D_1/D : $\quad D_1/D \preceq 1$; the lattice calculations [20] at $T = 0$ yield $\frac{D_1(u)}{D(u)} \preceq \frac{1}{3}, u > 0.1 fm$.

Therefore it seems reasonable to neglect D_1^E and D_1^B (note that $D_1^E = D_1^B = D_1$ at $T = 0$) and having in mind that $D^E = D^B$ at $T = 0$, one has an estimate

$$\eta \cong 1/2 \tag{61}$$

As another option with $D^E(0) = D_1^E(0) = D^B(0) = D_1^B(0)$, one can choose $\eta = 1/4$.

We have neglected in (59) i) higher order terms in g^n, ii) higher order cumulants $\ll \varphi^n \gg$ with $n \succeq 2$ and iii) the influence of the term Ω (24) and (43). As we discussed above, we believe that i) α_s is not large, $\alpha_s \preceq 0.5$ at all temperatures ii) higher cumulants are suppesed as $0(1/N_c^2)$, iii) from the structure of (24) and (43) one can notice that $\Omega \neq 1$ tends to decrease $| F |$ and since in the equilibrium the phase chooses the minimal F (maximal $| F |$) it will choose by that $\Omega = 1$. Note also, that studies of F for constant B_4 [25] seem to support our conclusion (in lowest orders graphs).

Of course our argument here is very qualitative and can be used only for a rough estimate of T_c. A more detailed study of the role of Ω is now in progress [26]. We now can plot both curves F_{high} (59), F_{low} (52) or rather the pressure $P = -F/V_3$ as a function of temperature and use again the principle of minimal F (maximal P). One can see in Fig.4 that there exists indeed some critical temperature $T = T_c$ and the system prefers to be in the low phase at $T < T_c$ and in the high phase at $T > T_c$. The transition is of the first order, and T_c can be computed equalizing (59) and (52)

In the leading order $0(N_c^2)$ one obtains:

$$T_c = (\eta \cdot \frac{45}{\pi^2} \cdot \frac{11N_c}{N_c^2-1}\frac{\alpha_s}{96\pi} < G^2 >)^{1/4} \tag{62}$$

We note that T_c is $0(N_c^0)$ which is reasonable since strong interactions (except for baryons) have finite limit both in masses and the range of interaction when $N_c \to \infty$. With the standard value [16].

$$G_2 \equiv \frac{\alpha_s}{\pi} < G^2 >= 0.012GeV^4 \tag{63}$$

we have for $\eta = 1/2$

$$T_c = (0.196\eta G_2)^{1/4} = 0.185Gev \tag{64}$$

However number of flavours $n_f = 0$ gluonic gluonic condensate should be 2-3 times larger [16], which yields $T_c = 0.22 - 0.24GeV$ and agrees well with the lattice calculations for gluodynamics [9].

In the next order $0(N_c)$ we take into account the quark contribution and still disregard all meson and glueball contributions (as well as nonideal gas corrections for gluons and quarks). We obtain

$$T_c = \eta^{1/4}(\frac{\frac{11}{3}N_c\frac{\alpha_s<G^2>}{32\pi}}{\frac{\pi^2}{45}(N_c^2-1)+\frac{7\pi^2}{180}N_c n_f})^{1/4} \tag{65}$$

For the choice (61) and $n_f = 2$ we have $T_c = 0.15GeV$ while for $n_f = 4$ we have $T_c = 0.134Gev$, which agrees well with recent lattice calculations [9,10] T_c^{QCD}(lattice)=$0.14\sigma GeV$ and $0.131GeV$ respectively.

7 Conclusion

We have started in the paper the background field formalism for $T > 0$ with the purpose to take into account the NP contributions in a systematic way.

The perturbation theory was formulated where expansion is of two different kinds: the cluster expansion for nonperturbative interaction and the usual expansion in powers of the coupling constant for perturbative quarks and gluons in the strong NP background.

In this first paper we concentrated on the lowest order terms: i) the gluonic condensate in the scale anomaly term ii) gluonic and quark loop diagrams in the background field.

We have shown in the previous Section that already a rough approximtion, where only terms i) and ii) are kept, yields a reasonable picture of the deconfining phase transition where a) the order of the transition b) the numerical value of T_c c) existence of non-perturbative effects at $T > T_c$, are correctly predicted. We also demonstrated that the terms i) and ii) are leading in the large N_c limit and the resulting value of T_c doesn't depend on N_c in this limit.

We plan to expand the formalism to include higher order terms and check the stability of our predictions. First of all, the role of gluonic (adjoint) Polyakov line Ω is being clarified [26]. This role is especially important for the question of the latent heat, which comes out too large with $\Omega = 1$ as compared to lattice calculations.

Another important possibility of the presented formalism is the study of infrared singularities of the free energy for $T > T_c$ connected to difficulty with the magnetic mass [3,13].

In the vaccum background field of QCD the confinement at $T > T_c$ is still present in the special planes [19] which cures the infrared divergencies and can solve in principle the problem of magnetic mass. This will be discussed in detail in a subsequent paper. Note added: after the original version of this manuscript has been published as a preprint [27], the author became aware of a similar estimate of T_c in [28], where it was assumed that no nonperturbative configurations are present for $T > T_c$ in the high temperature phase. This corresponds to $\eta = 1$ and results in $\sim 25\%$ higher values of T_c as compared to ours.

However in the pictures of [28] it is difficult to explain lattice results of [21-22] which require nonperturbative magnetic configuration for $T > T_c$.

The author is grateful for discussions to the members of the theoretical seminar of ITEP. A part of this work has been done while the author was a guest of the Max-Planck Institut für Kernphysik in Heidelberg. It is a pleasure for him to thank Professor H.A.Weidenmueller and all the staff of the Insitute for a kind hospitality. Fruitful discussions with H.G.Dosch, H.Leutwyler, P.Minkowski and H.D.Pirner are gratefully acknowleged.

APPENDIX

Calculation of the free energy of a noninteracting gluon loop via the path integral, Eq.(20).

The square brackets in (20) yield $(\frac{1}{2}\cdot 4-1)(N_c^2-1)=(N_c^2-1)$. The integral $(Dz)^w_{xx}$ can be calculated as the integral in $d\zeta(m)$ (see Eq.(15)) and using also (16) we get

$$\frac{\varphi(B=0)}{N_c^2-1}=V_3\beta\int_0^\infty\frac{dt}{t}\zeta(t)e^{-tp^2}dp_4\frac{d^3p}{(2\pi)^3}\sum_k\delta(p_4\beta-2\pi k)= \qquad (A1.1)$$

$$=\sum_k\int\frac{d}{ds}(\frac{M^2}{\vec{p}^2+(2\pi kT)^2})^s\mid_{s=0}\frac{V_3d^3p}{(2\pi)^3}=\sum_k\frac{V_3d^3p}{(2\pi)^3}ln\frac{M^2}{\vec{p}^2+(2\pi kT)^2}= \qquad (A1.2)$$

$$=\sum_{k=0,\pm1,\dots}\frac{V_3d^3p}{(2\pi)^3}[-\int_1^{\beta^2p^2}\frac{d\Theta^2}{\Theta^2+(2\pi k)^2}-ln(1+(2\pi k)^2)+lnM^2\beta^2] \qquad (A1.3)$$

Using a relation [1]

$$\sum_n\frac{1}{n^2+(\frac{z}{2\pi})^2}=\frac{2\pi^2}{z}(1+\frac{2}{e^z-1}) \qquad (A1.4)$$

we obtain

$$\frac{\varphi}{N_c^2-1}=-\int\frac{V_3d^3p}{(2\pi)^3}\int_1^{\beta|\vec{p}|}d\Theta(1+\frac{2}{e^\Theta-1})+inf.constants=$$

$$-\int\frac{V_3d^3p}{(2\pi)^3}[\beta p+2ln(1-e^{-\beta p})+const]= \qquad (A1.5)$$

$$=\frac{\beta}{3\pi^2}\int_0^\infty\frac{p^3dpV_3}{e^{\beta p}-1}=\frac{\pi^2T^3}{45};\quad \text{or}\quad F_0=-(N_c^2-1)\frac{V_3T^4\pi^2}{45}$$

FIGURE CAPTIONS

Fig.1. The gluon loop diagram in the background field B_μ (the first term on the r.h.s. of Eq.(10)).

Fig. 2. The ghost loop diagram in the background field B_μ (the second term on the r.h.s. of Eq. (10)).

Fig. 3. The gluon loop with the confining film as a result of averaging over background field - Eq.(27).

Fig. 4. Paths of integration $z(\tau)$ and $z'(\tau)$ in Eq.(27) in the in the confining regime.

Fig.5. The pressure P_{low} (corresponds to F_{low} Eq.(52)) as a function of temperature - dash- dotted line. The pressure P_{high} corresponding to F_{high}, eq.(59)) with the quark contribution -solid line, without it - dashed line. Transition temperatures $T_c(q)$ and T_c obtained with and without quark contribution respectively.

References

[1] J.Kapusta, Finite Temperature Field Theory, Cambridge Univ. Press, 1989.
L.McLerran, Rev. Mod. Phys. **58** (1986) 1021.

[2] R.Jackiw and S. Templeton, Phys. Rev.**D23** (1981) 22
E.Braaten and R.Pisarski, Phys. Rev. Lett. **64** (1990) 1338, Nucl. Phys. **B339** (1990) 310, ibid **B 357** (1990) 569.

[3] A.D.Linde, Phys. Lett. **96B** (1980) 289.

[4] B.S.De Witt, Phys. Rev. **162** (1967) 1195, 1239.
J.Honerkamp, Nucl. Phys. **B36** (1971) 130, **B48** (1972) 269
G.t'Hooft, Nucl. Phys.**62** (1973) 444
L. Abbot, Nucl. Phys. **B185** (1981) 189.

[5] N.G.van Kampen, Phys.Rep. **C24**, (1976) 171
H.G.Dosch, Phys. Lett.**B190** (1987) 177, H.G.Dosch and Yu.A.Simonov, Phys. Lett. **B205** (1988) 339.

[6] Yu.A.Simonov, Nucl. Phys. **B307**, (1988) 512 for earlier refs. see in Yu.A.Simonov, Yad.Fiz.**54** (1991) 224.

[7] Yu.A.Simonov, HP-THEP-93-16 JEPT Lett. **57** (1993) 525

[8] Yu.A.Simonov, Pis'ma JETF, **55**, (1992) 605

[9] S.Gottlieb et al. Phys.Rev.Lett. **59**, (1987) 1513
F.Karsch, HLRZ 92-62

[10] S.Gottlieb, Nucl. Phys. B (Proc. Suppl) 20, 247 (1991)

[11] Yu.A.Simonov, Nucl. Phys. **B324**, (1989) 67

[12] Yu.A.Simonov, Z. Phys. **53C**, (1992) 419

[13] D.J.Gross, R.D.Pisarski and L.G.Yaffe, Rev. Mod. Phys.**53**, (1981) 43

[14] R.Crewther, Phys.Rev.Lett. **28** (1972)1421; M.Chanowitz, J.Ellis, Phys. Lett. **B40** (1972)397; J.Collins, L.Duncan and S.Joglecar, Phys. Rev **D16** (1977) 438

[15] R.D.Pisarski, Phys. Rev.**29**, (1984) 1222

[16] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys.**B147** (1979)385,448

[17] W.Bardeen and V.I.Zakharov, Phys. Lett.**B91** (1980)111

[18] E.Witten, Nucl. Phys. **B156** (1979) 269.

[19] Yu.A.Simonov, Pis'ma JETF, **54**, (1991) 256

[20] M.Campostrini, A.Di.Giacomo and G.Mussardo, Z.Phys.**C25** (1984)179
A.Di Giacomo and H.Panagopoulos, Phys.Lett. **B 285** (1992) 133

[21] J.Polonyi and E.Manousakis, Phys Rev. Lett. **58**(1987) 847

[22] C.De Tar, J.Kogut, Phys. Rev. **36**, 2828(1987)
S.Gottlieb et al. Phys.Rev.Lett. **59**, (1990) 1881
T.De Grand et al.,Phys.Rev.Lett. **68**, (1992) 2125.

[23] M.Campostrini, A.Di.Giacomo et al. Phys.Lett. **B197** (1987)403

[24] N.A.Agasyan, Pis'ma JETF **57** (1993) 200

[25] N.Weiss, Phys. Rev. **D24** (1981) 475; V.M.Belyaev, IC/89/321.

[26] H.G.Dosch, H.J.Pirner and Yu.A.Simonov (in preparation).

[27] Yu.A.Simonov, preprint ITEP–3–93.

[28] P.Minkowski, Chech. J. Phys. **40** (1990) 1003